Chapter 7
Single-Sideband Modulation (SSB) and Frequency Translation

Contents

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Remember that an AM signal

$$s(t) = A_c[1 + k_a m(t)] \cos \omega_c t$$

has the Fourier transform

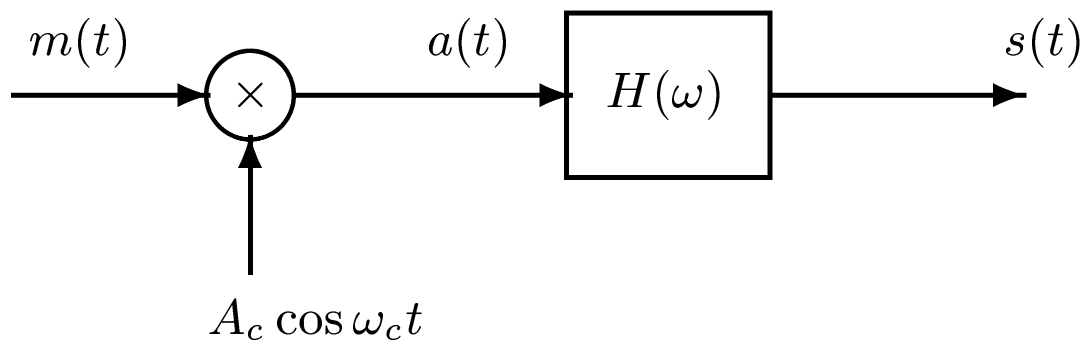
$$\begin{aligned} S(\omega) &= A_c \pi \delta(\omega + \omega_c) + A_c \pi \delta(\omega - \omega_c) \\ &\quad + \frac{A_c}{2} k_a M(\omega + \omega_c) + \frac{A_c}{2} k_a M(\omega - \omega_c) \end{aligned}$$

- The spectral components in the AM signal equal distances above and below the carrier frequency contain identical information because they are complex conjugates of each other.
- The portion above the carrier frequency is called the *upper sideband* and the portion below the *lower sideband.*

SSB Modulation by DSBSC-AM and Filtering

- In *single-sideband* (SSB) modulation only the upper sideband or the lower sideband is transmitted. Thus, SSB modulation requires half the bandwidth of AM or DSBSC-AM modulation.

- We will assume that the baseband message signal $m(t)$ is band limited with a cutoff frequency W which is less than the carrier frequency ω_c. Then the required channel bandwidth for an SSB signal is W.



SSB Modulator Using DSBSC-AM and Filtering

SSB Modulation by DSBSC-AM and Filtering (cont.)

First the DSBSC-AM signal

$$a(t) = A_c m(t) \cos \omega_c t$$

is formed which has the transform

$$A(\omega) = 0.5 A_c M(\omega - \omega_c) + 0.5 A_c M(\omega + \omega_c)$$

and is centered at the carrier frequency ω_c.

Then $H(\omega)$ selects the desired sideband. Upper sideband modulation uses the highpass filter

$$H_u(\omega) = \begin{cases} 1 & \text{for } \ |\omega| > \omega_c \\ 0 & \text{elsewhere} \end{cases}$$

and the lower sideband SSB modulation uses the lowpass filter

$$H_\ell(\omega) = \begin{cases} 1 & \text{for } \ |\omega| < \omega_c \\ 0 & \text{elsewhere} \end{cases}$$

Representing SSB Signals in Terms of Hilbert Transforms

Let the baseband message be $m(t)$ and its Hilbert transform $\hat{m}(t)$. The pre-envelope of the SSB signal has the transform

$$\begin{aligned} S_+(\omega) &= 2S(\omega)u(\omega) \\ &= 2A(\omega)H(\omega)u(\omega) \\ &= A_c M(\omega-\omega_c)H(\omega) \end{aligned}$$

and the transform of its complex envelope is

$$\tilde{S}(\omega) = S_+(\omega+\omega_c) = A_c M(\omega)H(\omega+\omega_c)$$

Upper Sideband Case

Substituting $H_u(\omega)$ for $H(\omega)$ gives

$$\begin{aligned} \tilde{S}(\omega) &= A_c M(\omega)u(\omega) = 0.5A_c M(\omega)(1+\operatorname{sign}\omega) \\ &= 0.5A_c M(\omega)[1+j(-j\operatorname{sign}\omega)] \\ &= 0.5A_c M(\omega) + j0.5A_c\hat{M}(\omega) \end{aligned}$$

Hilbert Transform Representation (cont.)

The complex envelope is

$$\tilde{s}(t) = 0.5A_c[m(t) + j\hat{m}(t)]$$

Therefore, the SSB signal can be expressed as

$$\begin{aligned} s(t) &= \Re e\{\tilde{s}(t)e^{j\omega_c t}\} \\ &= 0.5A_c\Re e\{[m(t) + j\hat{m}(t)]e^{j\omega_c t}\} \\ &= 0.5A_c m(t)\cos\omega_c t - 0.5A_c\hat{m}(t)\sin\omega_c t \end{aligned}$$

Lower Sideband Case

The transform of the complex envelope is

$$\begin{aligned} \tilde{S}(\omega) &= A_c M(\omega)u(-\omega) \\ &= 0.5A_c M(\omega)(1 - \operatorname{sign}\omega) \\ &= 0.5A_c M(\omega)[1 - j(-j\operatorname{sign}\omega)] \\ &= = 0.5A_c M(\omega) - j0.5A_c\hat{M}(\omega) \end{aligned}$$

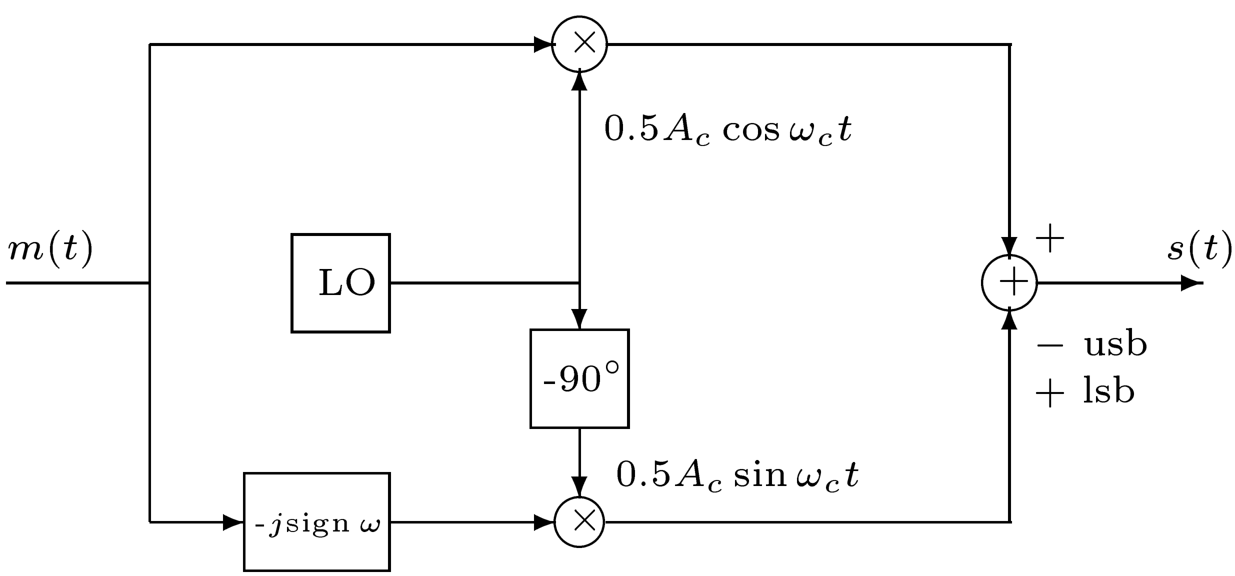
Therefore, the complex envelope is

$$\tilde{s}(t) = 0.5A_c[m(t) - j\hat{m}(t)]$$

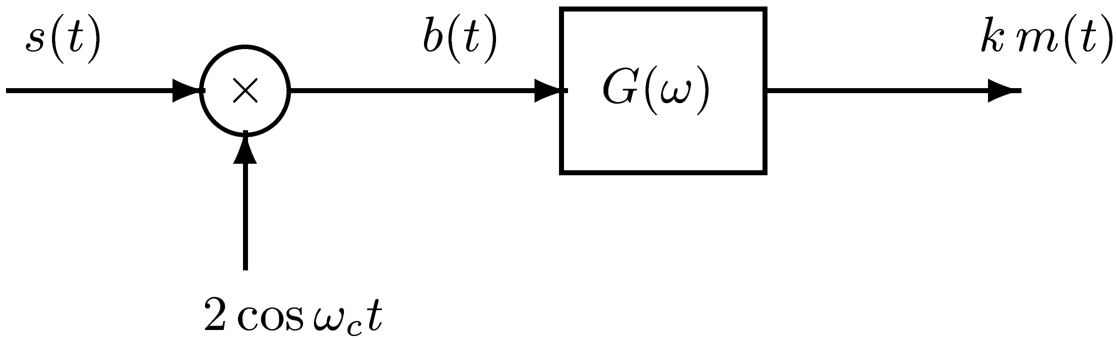
Single-Sideband Modulator Using a Hilbert Transform

The corresponding SSB signal is

$$
\begin{aligned}
s(t) &= \Re e\{\tilde{s}(t)e^{j\omega_c t}\} \\
&= 0.5A_c m(t)\cos\omega_c t + 0.5A_c\hat{m}(t)\sin\omega_c t
\end{aligned}
$$



Coherent Demodulation of SSB Signals



An SSB Demodulator

First the received signal is multiplied by a locally generated replica of the carrier signal. Multiplying the formulas for upper and lower sideband SSB signals by $2\cos\omega_c t$ yields

$$\begin{aligned} b(t) &= A_c m(t)\cos^2\omega_c t \mp A_c\hat{m}(t)\sin\omega_c t\cos\omega_c t \\ &= 0.5A_c m(t) + 0.5A_c m(t)\cos 2\omega_c t \\ &\qquad \mp 0.5A_c\hat{m}(t)\sin 2\omega_c t \end{aligned}$$

Coherent Demodulation (cont.)

Observe that

- $0.5A_c m(t)$ is the desired component.
- $0.5A_c m(t)\cos 2\omega_c t$ and $0.5A_c \hat{m}(t)\sin 2\omega_c t$ have spectra centered about $2\omega_c$.

The components around $2\omega_c$ are removed by the lowpass filter $G(\omega)$ with cutoff frequency W.

In practice, the demodulator shown above should be preceded by a *receive* bandpass filter that passes $s(t)$ and eliminates out-of-band noise.

Frequency Domain Analysis of Operation

Remember that $b(t) = s(t)2\cos\omega_c t$. So

$$B(\omega) = S(\omega + \omega_c) + S(\omega - \omega_c)$$

This translates the sidebands around $\pm\omega_c$ down to baseband and forms $M(\omega)$ which is the desired term and also translates them up to $\pm 2\omega_c$ which are the terms removed by the lowpass filter.

SSB Demodulator Using a Hilbert Transform

First take the Hilbert transform of $s(t)$ and form the pre-envelope

$$\begin{aligned} s_+(t) &= s(t) + j\hat{s}(t) = \tilde{s}(t)e^{j\omega_c t} \\ &= 0.5A_c[m(t) \pm j\hat{m}(t)]e^{j\omega_c t} \end{aligned}$$

where the plus sign is for upper sideband and the minus sign is for lower sideband modulation.

Multiplying the pre-envelope by $e^{-j\omega_c t}$ generates the complex envelope

$$\tilde{s}(t) = s_+(t)e^{-j\omega_c t} = 0.5A_c[m(t) \pm j\hat{m}(t)]$$

Taking the real part of the complex envelope gives

$$\begin{aligned} 0.5A_c m(t) &= \Re e\{s_+(t)e^{-j\omega_c t}\} \\ &= \Re e[s(t)+j\hat{s}(t)][\cos\omega_c t - j\sin\omega_c t] \\ &= s(t)\cos\omega_c t + \hat{s}(t)\sin\omega_c t \end{aligned}$$

which is proportional to the desired signal.

SSB Demodulator Using a Hilbert Transform (cont.)

This demodulator requires taking a Hilbert transform but does not require filtering out terms at twice the carrier frequency.

The modulator shown on Slide 7-6 is also a block diagram for a demodulator that implements the formula at the bottom of the previous slide if the input $m(t)$ is replaced by the received signal $s(t)$, the cosine and sine amplitudes are set to 1, and the plus sign is chosen at the output adder.

In practice, the demodulator would be preceded by an bandpass filter that passes the signal components and rejects out-of-band noise.

Need for a Pilot Tone

These two demodulators assume that the receiver has perfect knowledge of the received carrier frequency and phase. Unfortunately, this

SSB Demodulation (cont.)
Using a Pilot Tone

information cannot be derived by a system like the Costas loop because the SSB signal is the sum of an *inphase* component $m(t)\cos\omega_c t$ and a *quadrature* component $\hat{m}(t)\sin\omega_c t$.

A standard approach to solving this problem is to add a small sinusoidal component called a *pilot tone* whose frequency is not in the SSB signal band and has a known relationship to the carrier frequency. The pilot tone frequency is often chosen to be the carrier frequency when the baseband message signal has no DC components. The receiver can then generate a local carrier reference by using a narrow bandwidth bandpass filter to select the pilot tone and possibly following this filter by a phase-locked loop.

Frequency Translation

Reasons for needing frequency translation:

- To place the signal spectrum in an allocated channel.
- Several messages can be multiplexed together by shifting them to non-overlapping adjacent spectral bands and transmitting the sum of the resulting signals. This is called *frequency division multiplexing* (FDM).
- To correct for carrier frequency offsets caused by oscillator inaccuracies or Doppler shifts.

Let $s(t)$ be a bandpass signal with the frequency ω_0 somewhere in its passband. The problem is to translate the spectrum so that ω_0 is moved to $\omega_1 = \omega_0 + \Delta\omega$.

The first step is to form the pre-envelope

$$s_+(t) = s(t) + j\hat{s}(t)$$

Frequency Translation (cont. 1)

The corresponding Fourier transform is

$$S_+(\omega) = 2S(\omega)u(\omega)$$

The next step is to multiply by a complex exponential with frequency $\Delta\omega$ to get

$$\begin{aligned} r_+(t) &= s_+(t)e^{j\Delta\omega t} \\ &= [s(t) + j\hat{s}(t)][\cos\Delta\omega t + j\sin\Delta\omega t] \end{aligned}$$

which has the transform

$$R_+(\omega) = S_+(\omega - \Delta\omega)$$

This translates the original spectrum to the right by $\Delta\omega$ and moves the value at ω_0 to the frequency $\omega_1 = \omega_0 + \Delta\omega$.

Taking the real part of $r_+(t)$ gives the following formula for the translated signal:

$$r(t) = s(t)\cos\Delta\omega t - \hat{s}(t)\sin\Delta\omega t$$

Frequency Translation (cont. 2)

The real part of $r_+(t)$ can also be expressed as

$$r(t) = [r_+(t) + \bar{r}_+(t)]/2$$

so its Fourier transform is

$$\begin{aligned} R(\omega) &= [R_+(\omega) + \bar{R}_+(-\omega)]/2 \\ &= S(\omega - \Delta\omega)u(\omega - \Delta\omega) \\ &\quad + \bar{S}(-\omega - \Delta\omega)u(-\omega - \Delta\omega) \end{aligned}$$

The figure on Slide 7-6 is also the block diagram for a frequency translator if

1. the input $m(t)$ is replaced by the bandpass signal $s(t)$,
2. the frequency ω_c is replaced by $\Delta\omega$,
3. $0.5A_c$ is replaced by 1,
4. the negative sign is used at the output adder.

Frequency Translation (cont. 3)

Notice that the formula for computing $r(t)$ from $s(t)$ and $\hat{s}(t)$ above can be used even when the passband of the translated signal overlaps that of the original signal.

To do this using real signals would require a double conversion process where the signal is

1. first shifted to a non-overlapping band by multiplying by $\cos \omega_3 t$,
2. one sideband of this modulated signal is selected with a highpass filter,
3. and then another modulation is performed with the appropriate carrier frequency and the signal in the desired band is selected with a filter.

This is generally not as convenient for DSP applications.

Laboratory Experiments

Initialize the DSP and codec as in Chapter 2. Use a sampling rate of 8 kHz.

Making an SSB Modulator

- Write a program to implement the SSB modulator shown in Slide 7-6. Implement both the upper and lower sideband modulators. Take the message samples $m(nT)$ from the A/D converter. Send the modulated signal samples to the D/A converter. Implement the Hilbert transform FIR filter as an assembly language function that is called from C. Use the carrier frequency $f_c = 2$ kHz and an amplitude A_c that scales the output samples appropriately for the codec.
- Attach the signal generator to the EVM input and set it to generate a 1 kHz sine wave.

Making an SSB Modulator (cont. 1)

- Determine the theoretical formulas for the transmitted SSB signals for both the upper and lower sideband cases with the input $m(t) = A_m \cos 2\pi 1000\, t$.
- Observe the signals generated by your modulator on the scope for both cases and compare them with the theoretical ones.
- Vary the frequency of $m(t)$ from 0 to 2 kHz and observe $s(t)$ on the scope. Report what happens to the frequency of $s(t)$ for both the upper and lower sideband modulators.
- Next add a pilot tone $p(t) = A_p \cos 2\pi f_c t$ to the SSB output signal to provide a carrier reference for the demodulator you will make next. Use your judgement in choosing the value for A_p.

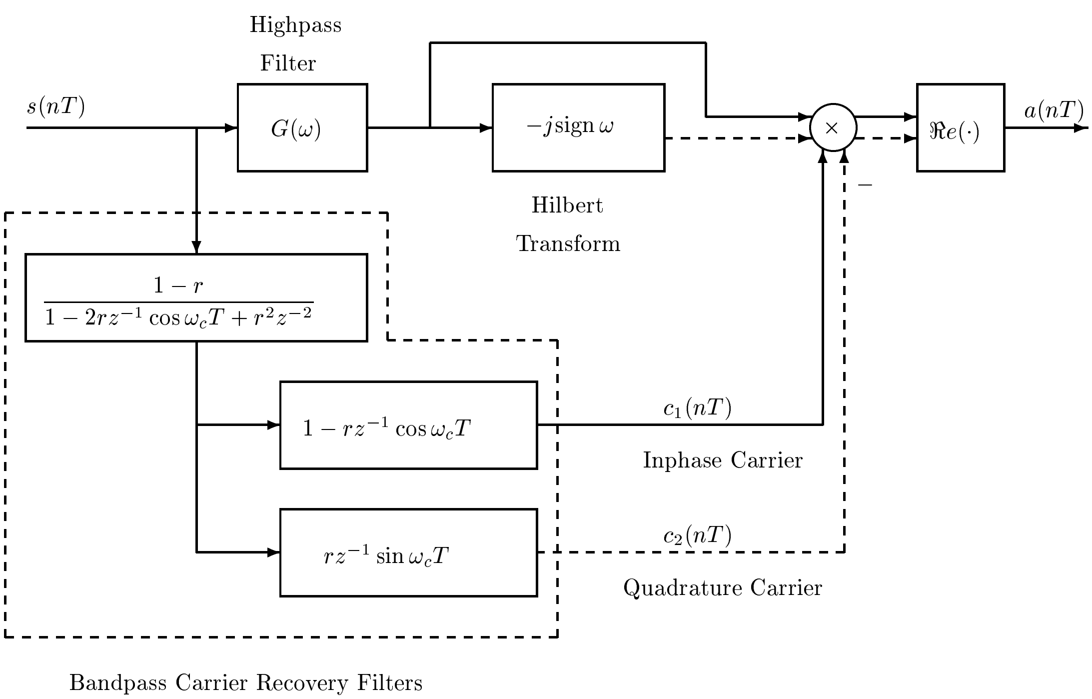
Coherent Demodulator for an SSB Signal

One possible demodulator is shown on Slide 7-19.

Extracting the Message Sideband

- In the upper sideband case, the input signal $s(nT)$ is passed through a highpass or bandpass filter $G(\omega)$ that rejects the pilot tone but passes the upper sideband.
- In the lower sideband case, $G(\omega)$ should be a lowpass or bandpass filter that passes the lower sideband and rejects the pilot tone.
- A notch filter could also be used to reject the pilot tone.
- In practice a bandpass receive filter would be used before the demodulator to reject out-of-band noise.

Block Diagram of a Demodulator



Using a Pilot Tone in SSB Demodulation

Coherent Demodulator (cont.)

Extracting the Pilot Tone

The portion enclosed by dotted lines is a pair of bandpass filters that extract replicas of the pilot tone and its -90° phase shift. The transfer functions of these two filters are

$$B_1(z) = \frac{(1-r)(1-rz^{-1}\cos\omega_c T)}{1-2rz^{-1}\cos\omega_c T + r^2 z^{-2}}$$

and

$$B_2(z) = \frac{(1-r)rz^{-1}\sin\omega_c T}{1-2rz^{-1}\cos\omega_c T + r^2 z^{-2}}$$

The denominators of these filters have the factorization

$$\begin{aligned} &1-2rz^{-1}\cos\omega_c T + r^2 z^{-2} \\ &\qquad = (1-re^{j\omega_c T}z^{-1})(1-re^{-j\omega_c T}z^{-1}) \end{aligned}$$

Thus the filter poles are at

$$z = re^{\pm j\omega_c T}$$

Coherent Demodulator (cont.)

The quantity r is a number slightly less than 1 and controls the bandwidth of the filters. The closer it is to 1, the narrower the bandwidth.

Theoretical Exercise

1. Prove that at the carrier frequency ω_c and when r is very close to 1, the transfer functions of the pilot tone extraction filters are approximately

   $$B_1(e^{j\omega_c T}) \simeq 0.5$$

   and

   $$B_2(e^{j\omega_c T}) \simeq -0.5j$$

2. By trial and error, choose a value of r that gives roughly a 50 Hz 3 dB bandwidth.

Coherent Demodulator (cont.)

Let the signal input to the multiplier be the pre-envelope $v(nT) = v_1(nT) + jv_2(nT)$. Then, the demodulator output is

$$\begin{aligned} a(nT) &= \Re e\{v(nT)[c_1(nT) - jc_2(nT)]\} \\ &= v_1(nT)c_1(nT) + v_2(nT)c_2(nT) \end{aligned}$$

Experimental Demodulator Exercises

Perform the following tasks:

1. Write a program to implement the demodulator discussed above and shown on Slide 7-19. Implement all FIR filters, like the Hilbert transform filter, by assembly language functions called from C. The filter, $G(\omega)$, can be an IIR filter.

Coherent Demodulator (cont.)

2. Now pipe the samples generated by your modulator program directly to your demodulator program internally within the DSP. Write the demodulator output samples to the D/A converter and check that it is working properly by observing the output on the oscilloscope.

3. Vary the message frequency and check that your modulator and demodulator are working correctly.

4. When your demodulator is working, send the modulator samples to the left channel D/A output. Connect the left channel analog output to the left channel input. Demodulate the left channel input and write the demodulated output samples to the right channel output. Observe the modulated signal (left channel output) and demodulated signal (right channel output) on the scope.