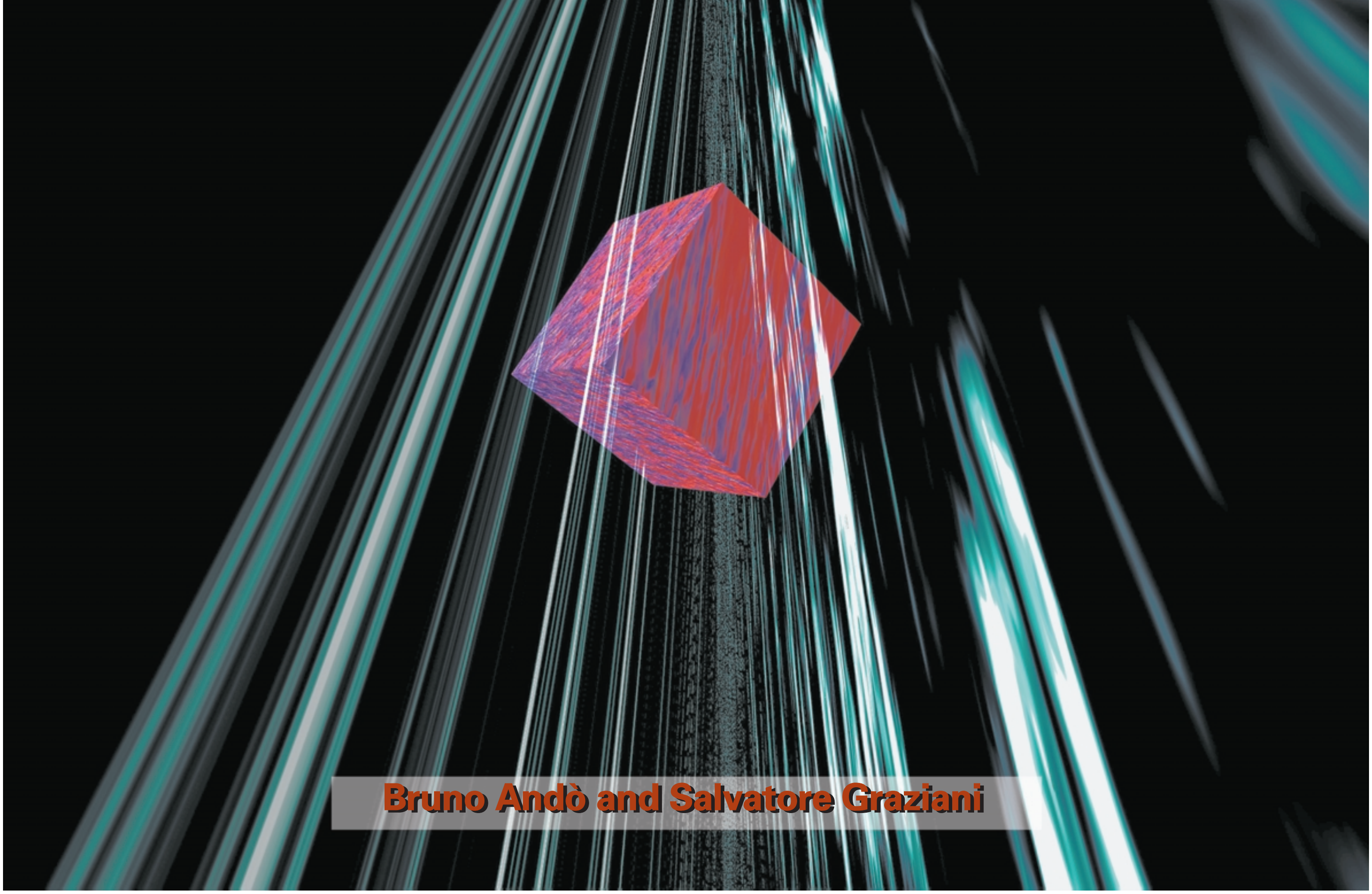

Bruno Andò and Salvatore Graziani



# Adding Noise to Improve Measurement

Certain complex systems can generate phenomena that classical theory cannot explain. Their behavior may be represented by a simplified scheme that combines both a deterministic and stochastic source [8], [12], [15]. To that end, researchers are using noise in various systems to enhance their function without altering the system.

## Noise-Added Systems

These studies focus on the behavior of bistable, threshold-based systems that are forced by both a periodic signal, with an amplitude lower than the system threshold, and a stochastic component. The interaction between the two signals transforms the system function at the same frequency of the periodic forcing signal. The phenomenon includes an increased signal-to-noise ratio and a peak in the output signal spectrum at the frequency of the forcing signal, corresponding to an optimal noise level. This is called *stochastic resonance* (SR) [9], [14].

The SR phenomenon can be interpreted as a reduction in threshold. Fig. 1(a) illustrates the case of a Schmitt trigger. SR



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is completely different from the resonance found in linear systems. The most obvious difference is the frequency of resonance; the resonance frequency in linear systems depends on the structural properties of the system, while the resonance frequency in SR is determined by the periodic signal.

*Dithering* is another noise-added technique normally used to enhance resolution in A/D converters by altering the quantization error spectrum [10], [18]. Although other techniques, such as oversampling, have been used to enhance the behavior of these devices, dithering has proven most effective when considering cost versus performance. By taking the concept to extremes, you can interpret the increase in the resolution of an A/D converter as linearizing its transfer function. Consequently, stochastic modulation techniques can linearize discrete systems. Unlike bistable systems, an improvement in the performance of quasilinear systems is achieved by modifying the shape of the transfer function.

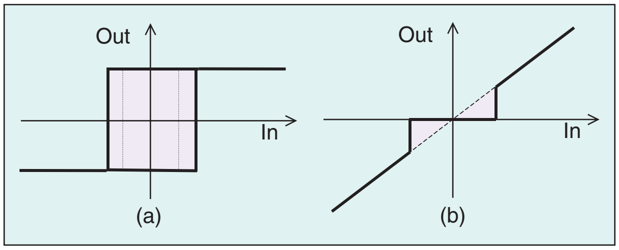


**Fig. 1.** (a) The system threshold (solid line) can be reduced (dotted line) by adding a suitable level of noise to the system. (b) A system with a linear I/O presents a threshold error in the proximity of null input signal amplitudes. The solid line represents this situation. The dotted line represents the ideal effect of noise linearization.

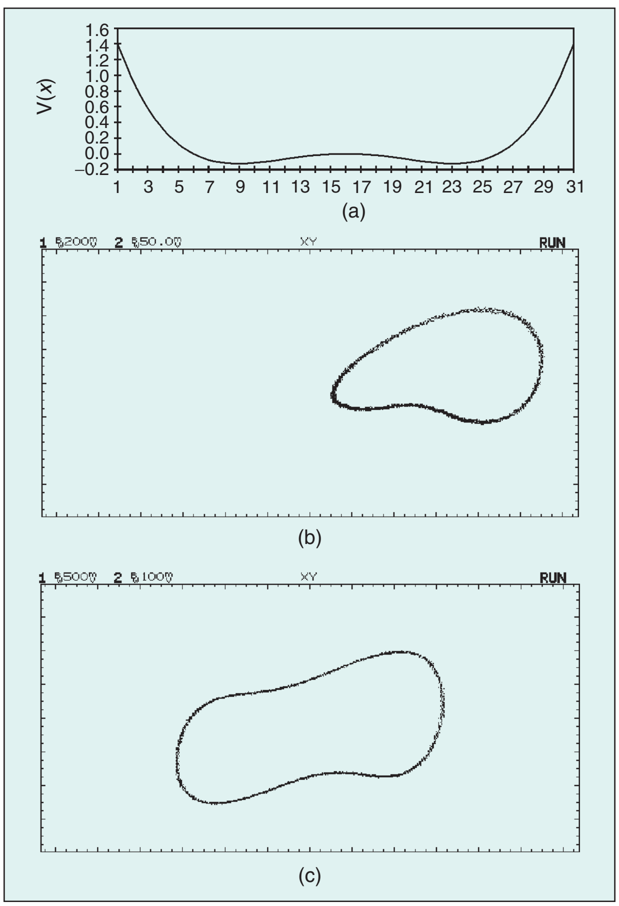


**Fig. 2.** (a) The quartic double well; (b) the state diagram when a suitable forcing signal larger than the threshold is applied; (c) the state diagram when a subthreshold forcing signal is applied.

In systems with two or more stable states, stochastic modulation techniques reduce the threshold, but the topology of the characteristic itself is left unaltered. In linear systems, on the other hand, action on the nonlinear areas involves changing the type of the original system.

Stochastic modulation can improve the performance of measurement devices, such as sensors affected by threshold error [3]. Fig. 1(b) illustrates an example of improved performance through stochastic modulation.

## Brownian Motion

A Brownian System is an example of noise-added theory applied to various natural phenomena and can model some physical devices [11], [14]. Several studies have emphasized the features of this phenomenon and its complexity arising from sensitivity to structural parameters [17]. Understanding how to optimize the behavior of this type of system in the presence of noise represents a significant advance in the field of noise-added systems. Brownian motion describes the movement of a particle that is subjected to collisions and other forces in a fluid. Macroscopically, the position $x(t)$ of the particle can be modeled as a potential $V(x)$ that is subjected to fluctuation and dissipation along with a deterministic forcing term. A Brownian system exhibits very interesting behavior, SR, when $V(x)$ is a quartic double well potential, as shown in Fig. 2(a) [11].

If the deterministic forcing term is set to zero (the particle is only subjected to fluctuation forces), transitions between the

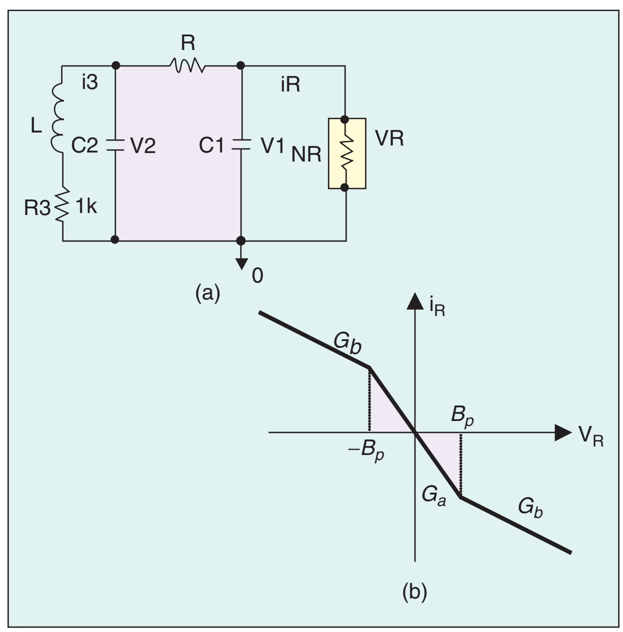


**Fig. 3.** (a) The unfolded Chua circuit; (b) the characteristic of the nonlinear resistor.

two potential wells occur at a rate given by *Kramers Rate* ($R_K$) and they depend on both the noise amplitude and some system parameters [11].

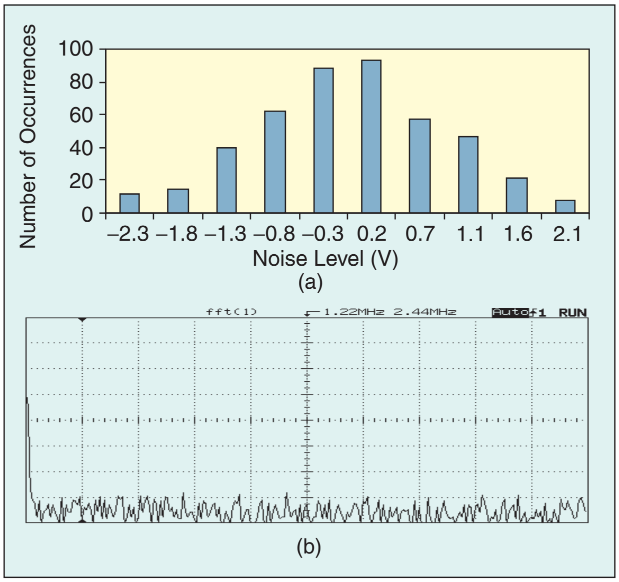


***Fig. 4.*** Data density plot and FFT of a Gaussian-like signal generated by the Chua circuit.

In the presence of a strong periodic forcing signal, with frequency $\nu=1/T$, the potential tilts back and forth, alternately raising and lowering the potential barrier of the right and the left wells. As a result, the particle rolls periodically from one potential well into the other one with the same frequency as the forcing signal. The state behavior of this system is shown in Fig. 2(b).

In the presence of a weak periodic signal, the particle should lie in a narrow range (attractor) of one state that is selected by the initial conditions, as illustrated in Fig. 2(c). In this case, a suitable noise signal could force the particle to roll from one potential well into the other. The periodic forcing synchronizes this transaction, which is the SR phenomenon.

This discussion implies that the performance of some systems may improve by adding a suitable quantity of noise. Furthermore, these techniques may optimize the behavior of systems operating in noisy environments by adapting and designing the system so that performance improves in the presence of noisy sources with well-defined characteristics. The remainder of the article will cover some new results that exploit the noise generation and the optimal noise tuning in noise-added systems.

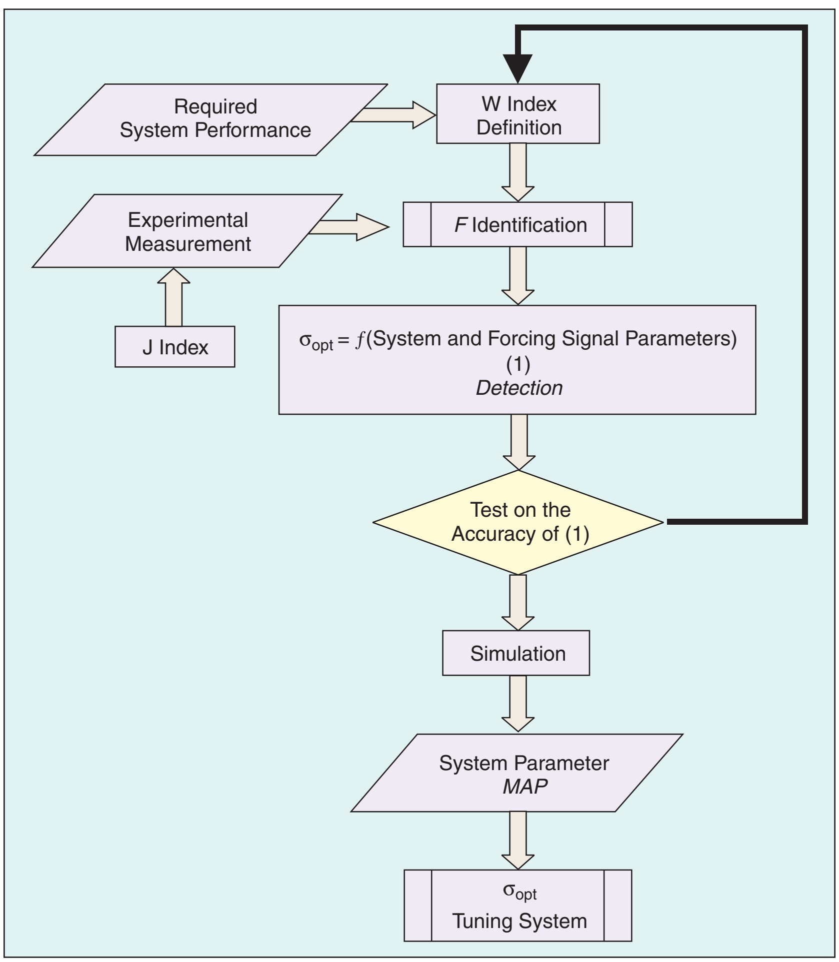


***Fig. 5.*** A schematic representation for noise tuning. It is valid for any class of system and allows for the analytical identification of the relationship between optimal noise variance and system parameters.

## Dithering versus SR

What is the difference between stochastic resonance and dithering? They are both noise-added techniques; they both enhance the performance of a system by reducing the threshold error; and they both require the optimization of certain parameters. So why use two names? Stochastic resonance is a phenomenon that is observed in the natural world, whereas dithering is an artificial technique. Stochastic resonance is a condition peculiar to bistable systems; it can reduce the system threshold (and, thus, render the system sensitive to forcing signals with an amplitude lower than the threshold) without altering the system's characteristics (i.e., the system remains bistable). Dithering, on the other hand, can increase the resolution of systems that have specific nonlinearities. Dithering linearizes the transfer function of the system near the nonlinearity, thus modifying the nature of the system (a system that was bistable, at least in the part affected by nonlinearity, is linearized). Therefore, reducing the threshold of a system is quite different from linearizing a part of the transfer function.

Dithering works on systems that are static around the set point. Optimization does not take into account the system's time constants. Stochastic resonance, on the other hand, is a phenomenon typical of both static systems (e.g., a trigger), where it is important to consider the forcing signal frequency, and dynamic systems (e.g., Brownian motion), where the system's time constants also play a role. (Recall the matching condition, where the characteristic frequencies of the system appear.)

## Noise Generation

The study of noise-added systems requires the use of a suitable noise generator. Sometimes laboratory equipment can be dedicated to generating noise. Other times, applications require a simple, low-cost generator.

The *Chua* circuit is a simple, well-known nonlinear system for generating signals with complex dynamics [13]. By varying its parameters, it can produce completely different behaviors, ranging from periodic to chaotic. For our needs in studying SR, it can provide a stochastic signal with desirable properties [7].

Fig. 3 illustrates a Chua circuit and the characteristic of its nonlinear resistor. Fig. 4 shows an example data density plot and the FFT of a Gaussian-like signal experimentally obtained by using this circuit [5], [13], [16].

## Noise Tuning

Identifying the optimal standard deviation, $\sigma_{opt,}$ for noise remains the main problem in dealing with noise-added systems. Generally, maximizing the signal-to-noise ratio will provide the appropriate value of $\sigma_{opt}$ [9].

The drawbacks of this approach are:

- The performance of the system cannot be assured. For example, the duty cycle of the output signal in a bistable system can not be fixed by setting the noise amplitude.
- Constraints on the system behavior (e.g., output fluctuation in sensors) cannot be taken into account. Noise-modulation techniques reliably reduce the physical threshold of some electronic devices (comparators, sensors), but raise the minimum noise level.
- When system parameter fluctuations occur, noise amplitude tuning is very complex, and real-time control of the variance value is not possible.
- The influence of external parameters on system performance cannot be taken into account.

An optimization of a more general index than the signal-to-noise ratio can counter these drawbacks to detecting the optimal noise variance [1], [2]. The index should include system performance (e.g., system switching times, its output duty cycle) and constraints (e.g., allowed output fluctuation).

Fig. 5 gives a schematic representation of this general approach. The main idea is to define, for each class of systems, a function *W* to be optimized; the signal-to-noise ratio being but one of the options. For stochastic systems, the analytical optimization of the index *W* will lead to a general relationship between the system parameters and the optimal standard deviation, $\sigma_{opt}$, that represents the optimal modulation for the noise level [2].

A probabilistic approach can obtain both a suitable form of the noise modulation and a map giving $\sigma_{opt}$ as a function of the system parameter [2]. An algorithm generates the map that screens the parameters of amplitude, *A*, and frequency, $\nu$, and obtains the corresponding $\sigma_{opt}$ values.

The deductive process, which gives rise to this procedure, is of general validity and can be applied to quasilinear systems. The main differences can be summarized as follows:

- Quasilinear devices are less affected by parametric variations, and implementation of the control strategy is less complex than bistable systems.
- Quasilinear systems require a complex post-processing stage. In bistable systems there is no post-processing stage because such systems naturally filter the high-frequency components. In quasilinear systems, on the other hand, this section is necessary to ensure that the undesir-

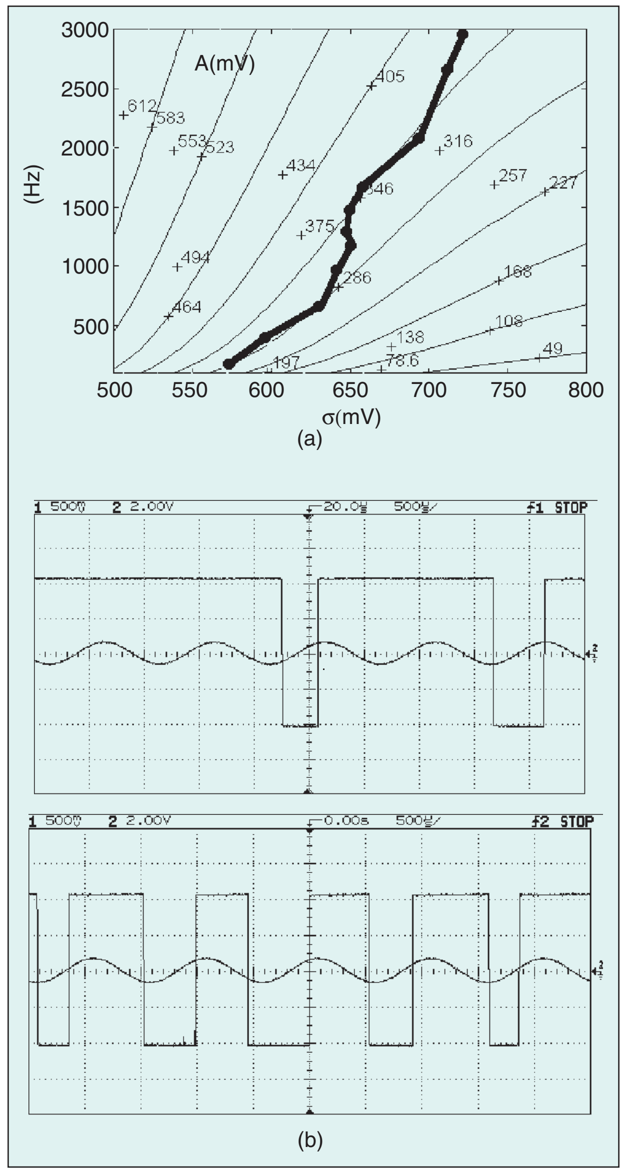


*Fig. 6.* (a) The iso-amplitude map; (b) the trigger output for two noise standard deviation values ($\sigma$ = 515 mV and $\sigma$ = 530 mV) due to a forcing signal with a frequency of 1 KHz and amplitude 500 mV.

able noise components that the system transfers to the output will be filtered out.

## Applications

We have given an overview of noise techniques that improve system performance. Now, we present some applications: the optimization of noise tuning in a Schmitt Trigger, the threshold linearization of an infrared (IR) sensor, and the optimal parameter tuning of a Brownian system subjected to stochastic fluctuation.

### Threshold Reduction in a Schmitt Trigger

It should be noted that the Schmitt Trigger represents a prototype for several physical devices. In particular, it models the behavior of some two-state sensing devices for which the hysteresis has been included to avoid unsuitable output caused by undesirable input signal fluctuations. Changing the hysteresis width (the system threshold value) by forcing a suitable level of noise into the device rather than by changing its physical structure, is of great interest when the same device must operate in different ranges.

We experimentally validated the noise optimization for a bistable system by applying both a noise-forcing signal and a periodic signal whose amplitude, $A$, was lower than the system threshold. Fig. 6(a) shows the iso-amplitude curves for the experimental trials with a bistable Schmitt Trigger that had a threshold of 700 mV [6]. The black circles, in the heavy line, show the minimum values of the input signal amplitude, allowing for system commutations (changing from one region operation to another) and the corresponding minimum noise variance evaluated at each forcing frequency. The thick line divides the map into two zones. For each forcing frequency, the amplitude of the forcing signal that allows for system commutation must be sought in the left zone of the map; the corresponding reading in the variance axis gives the minimum noise variance value [6]. Fig. 6(b) shows a limit condition of commutation as an example.

We adopted a simple form of the control law for the noise-tuning system and obtained good agreement between the original and the estimated data [6]. Fig. 7 shows the setup of the experimental tuning system.

### IR Sensor for Measuring Distance

Sensors for distance measurement have widespread commercial applications. The balance between cost and performance are the main factors determining the market for these devices. Noise-added techniques can improve the performance of low-cost devices.

A good example is an IR optical sensor used for distance measurement. IR optical sensors generally comprise an emitter and a photo-detector (photodiode or phototransistor). By forcing the emitter with a deterministic signal or a continuous voltage, the intensity of the radiation generated is proportional to the amplitude of the forced signal. The wave reflected by the target hits the receiving phototransistor and its intensity basically depends on the distance of the target. By mea-

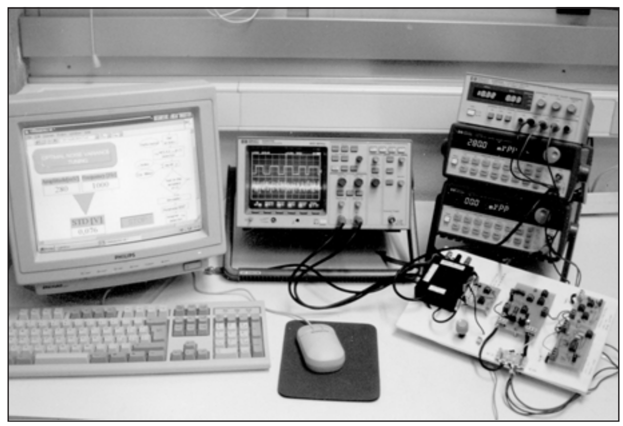

*Fig. 7.* Experimental setup of the noise tuning system for the Schmitt Trigger.

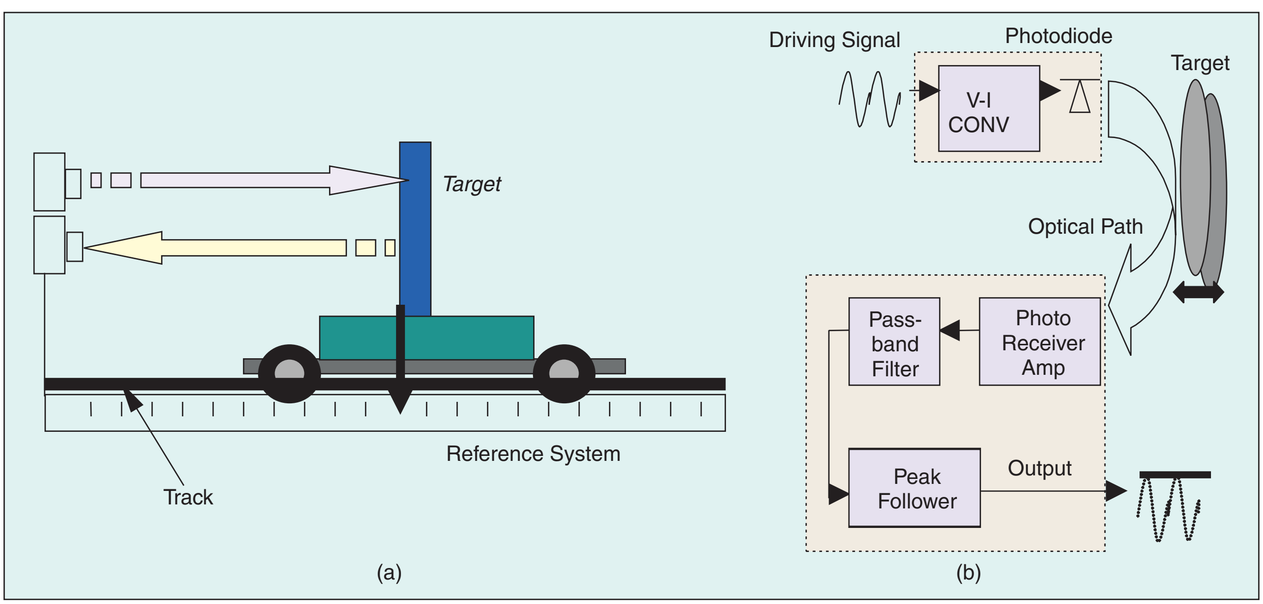


*Fig. 8.* Basic scheme outlining the functioning of the measurement system.

suring the intensity of the wave received, it is possible to calculate the distance of the target.

The setup illustrated in Fig. 8(a) tested noise modulation in an IR sensor. The measurement system is a trolley that moves freely along a track, a target mounted on the trolley, and an IR sensor. A reference index on the target indicates its distance from the sensor. The transmission and reception sensors are mounted at the ends of the track and lie on a surface parallel to the target. An amplification and a filtering stage are both required for conditioning the sensor output [Fig. 8(b)].

The target was an opaque, white reflecting surface. A forcing sinusoid with an amplitude of 6 Vpp and a frequency of 20 KHz characterized the transfer function of the system, as shown in Fig. 9(a). Without noise, the maximum range was 18 cm, which represents the physical threshold, $S_p$, of the system.

Then, we added uniform noise to the sinusoidal forcing signal and assessed the behavior of the IR sensor [3]. Fig. 10 shows some calibration curves for different values of the noise level. When stochastic signals are used, whose standard deviations are lower than the values shown in Fig. 10, the system behaves more or less as it does with no added noise. When the standard deviation is high, it causes excessive uncertainty in the measurements and saturation of the output signal.

Stochastic modulation increases the sensor's sensitivity in the outlying area of the field of measurement compared to nominal. Optimal values of the standard deviation improve the maximum measured distance from 18 to 24 cm [3].

Besides range, the uncertainty associated with measurements is important in assessing the performance of the sensor. Fig. 9(b) gives the calibration diagram for the noise-added IR sensor. We limited imprecision to $3\sigma$. The IR sensor, with and without added noise, gives comparable uncertainty bands.

## Parameter Optimization in QDW-Type Systems

Many applications must work with very-low-amplitude signals. Some devices can not easily process such signals due to both intrinsic threshold error and background noise. Threshold error desensitizes the device to low-amplitude signals while background noise makes it difficult to manipulate signals.

Let's consider a Brownian system with a threshold mechanism modeled by a quartic-double-well (QDW) potential presented in Fig. 2(a). This system is very interesting—it models several natural phenomena and physical devices, such as piezoelectric sensors and SQUIDD detectors. By shaping a QDW system instead of tuning the noise level, we can make a system sensitive to an underthreshold signal modulated by a stochastic signal. This is important in designing devices that operate in an environment where the characteristics of both the noise and the deterministic forcing signals are fixed by the application.

We change the structural parameters of a system to move it into the SR condition when it is forced by an unknown periodic signal modulated by white Gaussian noise with unknown statistical properties [4]. The optimal values of system parameters allow the system to switch with the same frequency as the forcing signal in the SR condition.

Since the characteristic of the forcing signal is unknown, analytical procedures for fixing the system parameters for SR and the best performance of the device are not straightforward. Hence, another index to be optimized is defined as follows:

- When all of the stochastic system parameters are fixed, the output power spectrum of the system will show a peak corresponding to the forcing frequency.
- This peak will be largest for optimal values of the system parameters, indicating SR.

The optimization procedure starts with the computation of a QDW system's output for a large set of values for the system parameters when the system is forced with the two signals. The algorithm computes the frequency corresponding to the peak value for each output spectrum, then it computes the dis-

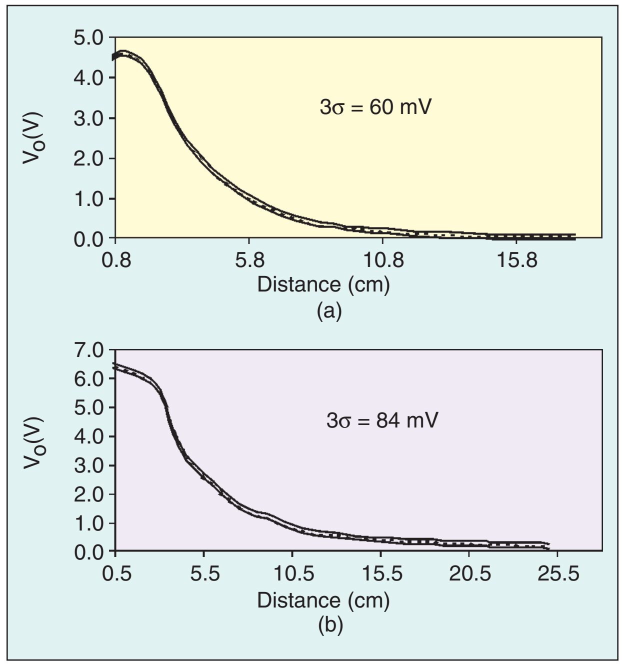


*Fig. 9.* Calibration curve of the IR sensor. (a) Without noise; (b) with noise.

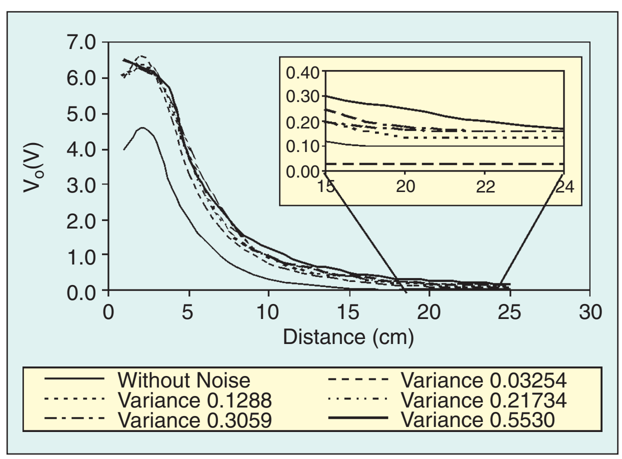


*Fig. 10.* Calibration curves at different values of the noise variance ($V^2$) when a uniform noise signal, generated by a digital waveform generator, is used.

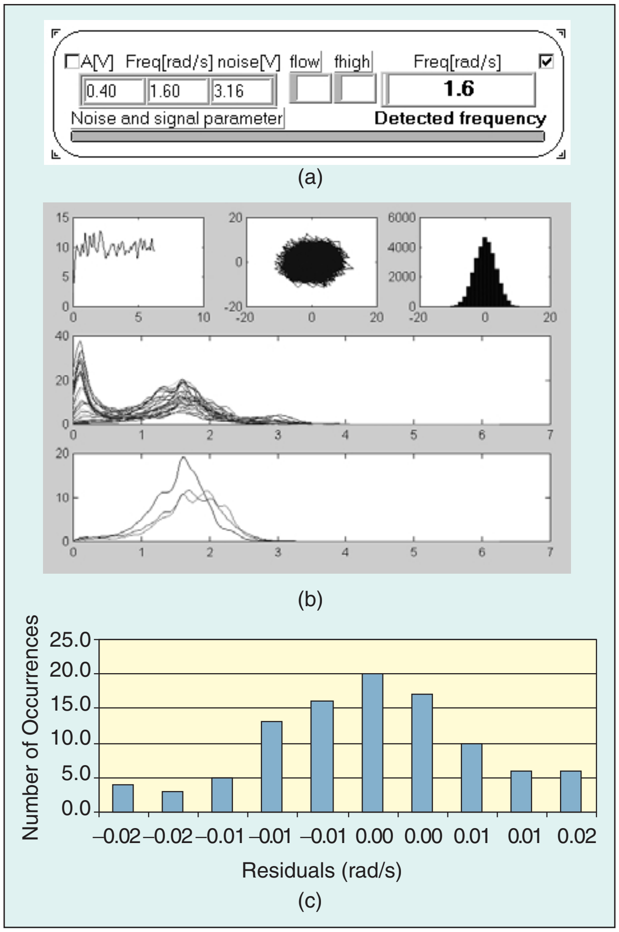


***Fig. 11.*** (a) Front panel of the virtual instrument for optimal system parameters setting, in case of $\omega_{x=1.6}$. (b) Data density plot of the residuals between the frequency of the deterministic forcing signal and the frequency detected by the algorithm.

tribution of these frequencies and chooses the frequency with the maximum number of occurrences. Experimentally, the maximum value is the forcing frequency, which confirms system sensitivity to very-low-amplitude (underthreshold) forcing signal.

We developed a virtual instrument in LabVIEW to simulate the selection algorithm (Fig. 11). The instrument is divided into two sections. The top section is dedicated to the input/output data actions while the bottom section is devoted to the graphic output. In the data input section, the user must insert the parameters of the periodic forcing signal, both amplitude and frequency, and the stochastic signal.

The top section of the graphic area shows the power spectrum, the correlation map, and the statistical distribution of the signal. This approach gives away no information on the forcing signal frequency. The middle section presents the spectra of the QDW system output when forced by the imposed signals (deterministic and stochastic) for a large set of the system parameters. The bottom section presents the spectra of the output showing the maximum in a narrow range of the frequencies selected by the algorithm. The top right dialog box shows the detected frequency; in the test mode, it can be compared with the input frequency given in the data section to verify the efficiency of the tool.

We have run a large number of trials to test the tool and found good performance. The data density plot of the residuals between the frequency detected by the algorithm and the frequency of the forced deterministic signal confirm the performance.

## Summary

We have shown the possibility of improving the performance of several system classes in the presence of noise. The strategy depends on the application. Stochastic resonance can reduce threshold while suitable noise modulation can linearize thresholds.

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