Steinhart-Hart Equation

Overall, the Steinhart-Hart equation has replaced the Beta equation as the most useful tool for interpolating the NTC thermistor R/T curve characteristic. The Steinhart-Hart equation is a third order polynomial which provides excellent curve fitting for specific temperature spans within the temperature range of -80 °C to 260 °C.

The Steinhart-Hart equation with the squared term eliminated is the most common form of the equation that is used and is usually found explicit in temperature T:

$$1/T = A + B\,(\ln R) + C\,(\ln R)^3$$

where: T = Kelvin units (°C + 273.15),

A, B, and C are curve-fitting coefficients, and

ln R = natural logarithm of a resistance in ohms.

To determine the A, B, and C coefficients for a particular temperature range, the resistance of an NTC thermistor is measured, under zero power conditions, at three temperature points, where T1 is the lowest temperature of the range, T2 is the mid temperature, and T3 is the highest temperature of the range.

The resistance/temperature data are placed into the following three equations.

$$1/T_1 = A + B(\ln R_1) + C(\ln R_1)^3$$

$$1/T_2 = A + B(\ln R_2) + C(\ln R_2)^3$$

$$1/T_3 = A + B(\ln R_3) + C(\ln R_3)^3$$

By solving the above three equations simultaneously, the coefficients A, B, and C are calculated from the following solutions:

$$C = \frac{x_3(y_1 - y_2) - x_1(y_1 - y_2) + y_1x_1 - y_1x_2 - y_3x_1 + y_3x_2}{(y_3)^3(y_1 - y_2) - (y_1)(y_2)^3 + (y_1)^4 - (y_1)^3(y_1 - y_2) + (y_3)(y_2)^3 - (y_3)(y_1)^3}$$

$$B = \frac{x_1 - x_2 + C(y_2)^3 - C(y_1)^3}{(y_1 - y_2)}$$

$$A = x_2 - By_2 - C(y_2)^3$$

Where:

$x_1 = 1/T_{low}$ $\quad$ $y_1 = \ln R_{T_{low}}$
$x_2 = 1/T_{mid}$ $\quad$ $y_2 = \ln R_{T_{mid}}$
$x_3 = 1/T_{high}$ $\quad$ $y_3 = \ln R_{T_{high}}$

T_i = Kelvin Temperature = t_i (˚C) + 273.15

T_{low} = Low temperature calibration point
T_{mid} = Mid temperature calibration point
T_{high} = High temperature calibration point
R_{T_i} = Resistance in ohms at temperature T_i

The Steinhart-Hart equation explicit in resistance is another useful form.

$$R_i = e(\exp)\left[\left[-\frac{1}{2}\left(\frac{A-\frac{1}{T_i}}{C}\right)+\left(\frac{1}{4}\left\langle\frac{A-\frac{1}{T_i}}{C}\right\rangle^{2}+\frac{1}{27}\left\langle\frac{B}{C}\right\rangle^{3}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}+\left[-\frac{1}{2}\left(\frac{A-\frac{1}{T_i}}{C}\right)-\left(\frac{1}{4}\left\langle\frac{A-\frac{1}{T_i}}{C}\right\rangle^{2}+\frac{1}{27}\left\langle\frac{B}{C}\right\rangle^{3}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}\right]$$

A simplified and more user-friendly version of the equation is as follows:

$$R = e(\exp)\left[\left(Y-\left\langle\frac{X}{2}\right\rangle\right)^{\frac{1}{3}}-\left(Y+\left\langle\frac{X}{2}\right\rangle\right)^{\frac{1}{3}}\right]$$

where

$$X = \frac{A-\frac{1}{T}}{C} \quad \text{and} \quad Y = \left(\left\langle\frac{B}{3C}\right\rangle^{3}+\frac{X^2}{4}\right)^{\frac{1}{2}}$$

As with any powerful tool, certain precautions need to be taken when using the Steinhart-Hart equation in order for the user to achieve the desired accuracy and uncertainty of the resistance vs. temperature data to be calculated. By understanding the strengths and limitations of the Steinhart-Hart equation, one can optimize the results for a particular application. Listed below are some guidelines which show the amount of interpolation error introduced by the equation for each of the following conditions, where the temperature span over which the R/T data to be calculated is defined by the end points t_{low} and t_{high} expressed in units of degrees Celsius (°C):

1) ≤ 0.003 °C error for 50 °C temperature spans within the range of temperatures (t)
   0 °C ≤ t ≤ 260 °C.

2) ≤ 0.02 °C error for 50 °C temperature spans within the range of temperatures (t)
   -80 °C ≤ t ≤ 0 °C.

3) ≤ 0.01 °C error for 100 °C temperature spans within the range of temperatures (t)
   0 °C ≤ t ≤ 260 °C.

4) ≤ 0.03 °C error for 100 °C temperature spans within the range of temperatures (t)
   -80 °C ≤ t ≤ 25 °C.

If an application requires a curve fit with the maximum possible accuracy over a temperature span wider than 50 °C or 100 °C, the desired temperature span can be broken down into 50 °C or 100 °C increments for calculation of the A, B, C coefficients and resistance vs. temperature data. The resistance ratio (R_t/R_{25}) vs. temperature tables published by Cornerstone Sensors were developed from Steinhart-Hart equation calculations performed over several 50 °C spans, such as -50 °C to 0 °C, 0 °C to 50 °C, 50 °C to 100 °C , and 100 °C to 150 °C.

Since the A, B, C coefficients are curve-fitting coefficients for the NTC thermistor curve characteristic, their values depend upon the type of material formulation, the accuracy or uncertainty of the temperature and resistance measurements, and the temperature span. Therefore, depending upon the user's temperature measurement capabilities and temperature span chosen, slight variations may occur when comparing the user's data to the published tables.

See also: A, B, C Coefficients Table

A, B, C Coefficients for Steinhart-Hart Equation

The table of A, B, and C coefficients listed below were derived from the following Steinhart-Hart equation explicit in T:

$1/T = A + B(\ln R_T) + C(\ln R_T)^3$

where

R_T = the resistance value in ohms at Kelvin temperature T.

The resistance ratio values included in the R/T Curve Manufacturing Tolerance Tables and the 1 °C Resistance Ratio vs. Temperature Tables found in the Technical Information Section were calculated by using the A, B, and C coefficients listed below in the following Steinhart-Hart equation explicit in R.

$$R = e(\exp)\left[\left(Y - \left\langle\frac{X}{2}\right\rangle\right)^{\frac{1}{3}} - \left(Y + \left\langle\frac{X}{2}\right\rangle\right)^{\frac{1}{3}}\right]$$

where

$$X = \frac{A - \frac{1}{T}}{C} \quad \text{and} \quad Y = \left(\left\langle\frac{B}{3C}\right\rangle^3 + \frac{X^2}{4}\right)^{\frac{1}{2}}$$

For an expanded version of the equation, refer the Steinhart-Hart Equation explanation found in the About Thermistors section.

Important Note: To optimize interpolation accuracy of the resistance vs. temperature data, the A, B, C coefficients of the Steinhart-Hart equation shown above were calculated specifically for the temperature spans and resistance values listed below. The Cornerstone Sensors resistance ratio vs. temperature tables were created by dividing the resistance value at each temperature by the appropriate resistance at 25 °C. For more information on how to use the equation to calculate data for other resistance values and/or temperature ranges, please contact the factory.

R/T Curve	Temperature Range (°C)	Coefficients for the Steinhart-Hart Equation		
		A	B	C
B	-50 to 0	1.439114856904070E-03	2.693066430764570E-04	1.653440958554570E-07
	0 to 50	1.440548920932420E-03	2.690725842060890E-04	1.661922621891600E-07
	50 to 125	1.440327988425520E-03	2.690459927836250E-04	1.679441362548790E-07
D	-50 to 0	1.133136854163360E-03	2.336519292538650E-04	8.849976714136810E-08
	0 to 50	1.124974037152450E-03	2.347653241229690E-04	8.546325084516770E-08
	50 to 100	1.119828875495430E-03	2.360897740308300E-04	7.508299550946710E-08
	100 to 150	1.120748323248730E-03	2.353531346746540E-04	8.909503408745950E-08
E	-50 to 0	9.329599574968520E-04	2.214235932652170E-04	1.263286697870110E-07
	0 to 50	9.327935342661280E-04	2.214507360140700E-04	1.262325823098370E-07
	50 to 100	9.315712556993570E-04	2.216946671543180E-04	1.249321433697330E-07
	100 to 150	9.266934080778390E-04	2.228124367891810E-04	1.167171733506130E-07
F	-50 to 0	1.028525291852400E-03	2.392327985577990E-04	1.562478971912460E-07
	0 to 50	1.029194767422500E-03	2.391275183977950E-04	1.566277149730310E-07
	50 to 100	1.028687651810930E-03	2.391866941635480E-04	1.566594211560540E-07
	100 to 150	1.026416673809340E-03	2.397397615551010E-04	1.518913935501530E-07
G	-40 to 0	8.630777018579910E-04	1.999086501945880E-04	1.244201049853700E-07
	0 to 50	8.436437679500710E-04	2.021008350641180E-04	1.203143775299850E-07
	50 to 100	8.331470500361880E-04	2.032624244467480E-04	1.188714921067930E-07
	100 to 150	8.356070132533520E-04	2.027581789531380E-04	1.217977002758560E-07
R	-50 to 0	8.312653413479670E-04	2.083082877047050E-04	8.162433729410980E-08
	0 to 50	8.274918692383500E-04	2.087547277229870E-04	8.069870440386500E-08
	50 to 100	8.252715954912970E-04	2.090759654020190E-04	7.970254981668580E-08
	100 to 150	8.169484569941060E-04	2.105821263402370E-04	7.242814459308220E-08
V	-50 to 0	1.912033993648160E-03	3.064159978007270E-04	2.861356543672190E-07
	0 to 50	1.938993181772380E-03	3.004098484042120E-04	3.234005399703990E-07
	50 to 100	1.953893905132090E-03	2.945030434912480E-04	4.609116815442370E-07
W	-50 to 0	1.328734000368390E-03	2.871918721144560E-04	1.257114168390270E-07
	0 to 50	1.328156656857180E-03	2.873061594285200E-04	1.250512619348640E-07
	50 to 100	1.300117357404990E-03	2.943269678705660E-04	5.985964560607700E-08